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No early warning signals for stochastic transitions: insights from large deviation theory

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In the study of Boettiger & Hastings [1], we demonstrated that conditioning on observing a purely stochastic transition from one stable basin to another could generate time-series trajectories that could be mistaken for an early warning signal of a critical transition (such as might be due to a fold bifurcation [2]), when instead the shift is merely due to chance. While the goal was to highlight a potential danger in mining historical records for patterns showing sudden shifts when seeking to test early warning techniques, Drake [3] draws attention to a potentially more interesting consequence of our analysis. Drake argues that the bias observed could be used to forecast purely stochastic transitions—a task previously thought to be impossible [4]. We feel this interpretation is too generous. The pattern Drake points to arises in any large deviation, regardless of whether a system is or is not at elevated risk for a transition. We illustrate this pattern in systems with and without bistability, demonstrating that early warning signals do not exist for purely stochastic transitions.

Here, we provide a numerical demonstration that the pattern in question for consideration of an early warning signal appears not only before purely stochastic transitions (as seen in [1]) but also during any large deviation. As large deviations can occur even in stochastic systems that have only a single stable point, these patterns cannot be considered indicators of stochastic transitions. We demonstrate this in two scenarios: first using the Allee model of alternative stable states considered in [1], (eqn 2.1–2.2 and fig. 2), and then in a simple Ornstein–Uhlenbeck (OU) model, which has only a single stable state. Rather than condition on a stochastic transition having occurred (as in [1]), we now condition on having merely observed a sufficiently large deviation (larger deviations will be rarer and show a more pronounced pattern). We pick values such that we obtain a sample of a few hundred large deviation events in a sample of 20 000 replicates.

The OU model is defined by a stochastic differential equation in which there is only a single optimum whose strength is proportional to the displacement,

$$dX_t = -\alpha X_t dt + \sigma dB_t,$$

where the state X_t oscillates around a stable point (at zero in these arbitrary units), driven by Brownian noise dB_t of intensity σ and restorative force α .

The analysis for each model proceeds exactly as in [1]: for each model, we generate 20 000 replicate time series. We condition upon only those experiencing a deviation of size L ($X \leq 250$ in the Allee model and $X \leq -4$ in the OU model). For the sequence of observations immediately leading up to the large deviation, we compute the warning signals of variance and autocorrelation over a sliding window of half the length of the time series, and we summarize the increasing or the decreasing trend observed in the variance and autocorrelation using Kendall's τ rank correlation coefficient (all following the method for early

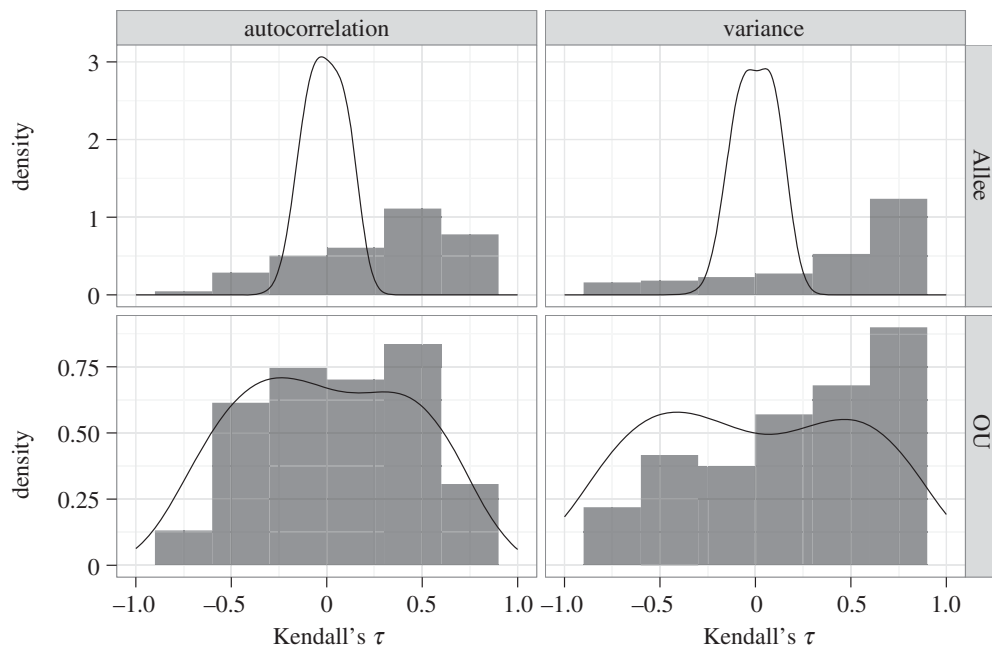


Figure 1. Histogram shows the frequency the correlation statistic τ observed for each warning signal (variance, autocorrelation coefficient) on the large deviation samples from each model. Background distribution of all samples show by smooth line (kernel density estimate). More positive values of τ are supposed to indicate a rising indicator which can be a signal of an approaching transition [2]. The OU model uses $\alpha = 5$, $\sigma = 3.5$, $t \in (0, 10)$, 2000 replicates, 20 000 sample points each. Conditionally selected trajectories experiencing a deviation of at least -4 , and analysed the 1500 data points prior to the threshold to determine a warning signal (following [5]). Code and data available in the Dryad repository: <http://dx.doi.org/10.5061/dryad.1dj62>.

warning indicators outlined in [5]). We repeat this analysis on the entire set of time series under each model to obtain null distributions for the τ statistic.

We find (figure 1) that τ is significantly skewed towards positive values when conditioning on large deviations in both models. This demonstrates that it is the presence of the large deviation, not the presence of the stochastic transition we condition on in the study of Boettiger & Hastings [1], that is responsible for this pattern (just as we claimed without example then).

Observing the bias shown in the figures here depends on having a rapid enough sample frequency to capture the escape trajectory and a long enough trajectory for the statistic to demonstrate an increase over time. Since large deviations owing to stochastic forces alone must be fast, so must be the accompanying warning signal and management response (which will show up on the timescale of the perturbation). Note that fast relative to the system dynamics may or may not be fast relative to the timescale of management (just as with bifurcation-driven warning signals [6]). The wider null distribution in the OU model results from the sample window being shorter relative to the system timescale.

One might consider this a corollary of the Prosecutor's Fallacy we originally presented, which demonstrated that examples of sudden transitions historically selected from the literature could be mistaken for positive evidence of early warning signals when they were in fact owing to purely stochastic transitions. Here, we have seen how any large deviation could be similarly misleading, whether or not it results in a stochastic transition to an alternative stable state. From a classical result of the large deviation theory, one can gain considerable intuition about why these chance deviations show much higher variance and autocorrelation than expected

from the stationary distribution of a stable point. Though large deviations are rare—the time we must wait to observe a deviation of size L in the system above scales as $\exp(L^2/\sigma^2)$ (the familiar Arrhenius relationship), when they occur it is very rapid. The expected time for an excursion to a distant point L that does not again cross the stable point before reaching L scales as $\log(L/\sigma)$, just as a trajectory returning down the gradient of the attractor from L to the stable point (proofs [7] or [8]). While most trajectories in the stationary distribution take steps in each direction with equal probability, these large deviations moving rapidly to the boundary will consequently show a greater autocorrelation. In achieving a much greater deviation than typically observed, these trajectories will also show an increase in variance, as observed. That such trajectories appear to be pulled in the direction of their escape rather than climbing away against a restorative force has led to confusion before. Lande [8] argues how this shows a 'punctuated equilibrium' pattern of stasis followed by rapid change that could arise entirely from small steps, and Drake & Griffen [9] empirically demonstrate this phenomenon in the trajectories of local population extinctions.

In conclusion, we heartily agree with the need for a decision-theoretic approach to early warning signal questions [10]. Central to a decision-theoretic approach is the enumerating alternative scenarios that are possible given the observed data. We have highlighted how purely stochastic transitions and large deviations are such possibilities. The challenge of sufficient or unique early warning indicators is not limited to stochastic shifts, but includes the more typical critical transitions. For instance, rising variance or autocorrelation patterns typical of fold bifurcations can be observed in more benign bifurcations or smooth transitions [11]. Early warning signals may offer a promising technique that

will one day allow us to avoid seemingly unpredictable catastrophes—but we must not lose sight of just how difficult are the challenges involved. A key step here and for early warning indicators more generally is to understand these other circumstances in which they can arise that we may then develop ways to eliminate those possibilities. Though we may never be able to detect purely stochastic transitions,

perhaps these approaches in this discussion may lead to more unique and sufficient indicators for true critical transitions.

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